Mixed Convective Burning of a Horizontal Flat Plate

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Chemically-reacting laminar boundary-layer flow over a horizontal fuel slab is studied, in the limit of flame-sheet combustion, for the entire range of mixed convection with the buoyancy force either aiding or opposing the development of the boundary layer. Rigorous solution as well as approximate solution assuming local similarity are obtained for the Shvab-Zel'dovich variables, velocity profiles, surface shear and temperature gradients, flame-front standoff distances, and excess pyrolyzates. Comparison of these results show that, while the approximate solution provides good agreement for the mass burning rate and the flame location, the velocity profiles deviate significantly from the rigorous solution in the regime of intense mixed convection. Complete solutions are also obtained for opposing flows to the point of separation for mixed-convective burning over both horizontal and vertical fuel surfaces.

Nomenclature

B = mass transfer number, $[QY_{O\infty}/(\nu_O M_O) - c_p(T_w - T_\infty)]/L$
c_p = specific heat
D = species diffusivity
f = reduced stream function
F = excess pyrolyzate
g = acceleration of gravity
Gr = Grashoff number, $g\beta(T_w - T_\infty)x^3/\nu_\infty^2$
h = specific enthalpy
ℓ = parameter, BL/h_w
L = effective heat of gasification
m = local mass flux
P = pressure
$\bar{P}$ = nondimensional pressure, $P/(\rho_\infty u_\infty^2)$
Pr = Prandtl number
Q = heat of combustion per gram of oxidizer
Re = Reynolds number, $u_\infty x/\nu_\infty$
T = temperature
$\bar{T}$ = normalized temperature, $(T - T_\infty)/(T_w - T_\infty)$
u, v = x and y velocities
W = molecular weight
x, y = streamwise and transverse directions
$\bar{y}_f$ = scaled flame standoff distance
Y = mass fraction
β = coefficient of thermal expansion
Γ_i = Shvab-Zel'dovich coupling function
$\bar{\Gamma}$ = normalized Shvab-Zel'dovich coupling function
δ = parameter, $(T_w - T_\infty)/T_\infty$
η = similarity variable
θ = scaled pressure
λ = thermal conductivity
μ = viscosity coefficient
ν = kinematic viscosity or stoichiometric coefficient
ζ_h = mixed convection ratio for horizontal case, $Gr/Re^{5/2}$
ζ_v = mixed convection ratio for vertical case, Gr/Re^2
ξ = mixed convection parameter, $(1+\zeta_h^2)^{-1}$ or $(1+\zeta_v^2)^{-1}$
ρ = density
σ = $(Y_{O\infty}\nu_F W_F)/(Y_{Fw}\nu_O W_O)$
τ_w = shear stress at the wall
ψ = stream function

Subscripts

f = flame
F = fuel
i = index for Shvab-Zel'dovich coupling function
O = oxygen
s = flow separation point
w = wall
∞ = ambient

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I. Introduction

BUOYANCY forces are capable of either reinforcing or retarding a forced convective flow when the induced density variations associated with temperature stratification within the boundary layer are large. The modifications are especially significant when chemical reactions are present within the boundary layer, as exemplified by such practical situations as those involving solid-propellant combustion and compartment fires.

Much is known about the pure forced and free convective burning over horizontal and vertical surfaces.[1-4] Studies on mixed convective heat transfer and burning, however, are frequently limited to situations involving either strongly forced convective burning with weak natural convective perturbations or strongly natural convective burning with weak forced convective perturbations.[5-8] The analytical difficulty here lies with the use of either Reynolds number or the Grashoff number for nondimensionalization, thereby rendering the solution singular in one of the limits.

Recently, when studying the self-similar mixed convective stagnation-point combustion, Fernandez-Pello and Law[9] reported that by using a unified parameter of the form $(Gr^2 + Re^4)^{1/8}$ for scaling, the resulting nondimensionalized equations become uniformly valid throughout the entire regime of mixed convective flow intensities. This parameter has subsequently been adopted by Fernandez-Pello and Pagni[10] in the study of mixed convective burning over vertical surfaces.

The mixed-mode parameter $(Gr^2 + Re^4)^{1/8}$ used for vertical surfaces is not applicable to horizontal surfaces because, as is well known,[4,11] free convective flows scale differently for these two situations. Thus, in the present study, we propose $(Gr^2 + Re^5)^{1/10}$ as the appropriate scaling parameter for mixed convective flows over horizontal surfaces. As will be subsequently demonstrated, the solution thus obtained again becomes uniformly valid over the entire range of the mixed convective flow. In particular, similarity solutions are recovered in the limits of pure forced and pure natural burning, as should be.

The formulation is presented in the next section, while the calculated results are discussed in Sec. III. Both the rigorous as well as the locally similar solutions are obtained. The flow structure for opposing flows up to the point of separation is also studied for both horizontal and vertical plates.

II. Formulation

Figure 1 shows the physical situation involving diffusional burning within the mixed convective laminar boundary-layer flow over a horizontal flat plate, whose temperature is maintained constant at T_w. The steady diffusion flame sheet is established through the stoichiometric reaction between the gasified fuel from the plate surface and the oxygen in the ambient gas stream. The chemical heat release at the flame is used to sustain gasification of the condensed fuel and to heat the fuel and oxidizer as they approach the flame.

At any streamwise location x, the fuel vapor present between the fuel and flame surfaces represent the amount of fuel, called excess pyrolyzate,[6] which has been gasified but has not yet reacted. Thus, if combustion is terminated there, this unreacted excess pyrolyzate represents either reduced combustion efficiency or a source of combustible vapor, depending on the situation of interest.

The gravity vector can point either downward or upward for situations involving aiding or opposing flows, respectively. Separation is expected in the later case.

With the usual assumptions associated with the formulation of chemically reacting boundary-layer flows,[12] the governing equations are:

Continuity

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \tag{1}$$

Momentum

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial P}{\partial x} \tag{2}$$

Γ_i conservation

$$\rho u\frac{\partial \Gamma_i}{\partial x}+\rho v\frac{\partial \Gamma_i}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\lambda}{c_p}\frac{\partial \Gamma_i}{\partial y}\right),\quad i=F,T \tag{3}$$

State

$$(\rho-\rho_\infty)=-\beta\rho(T-T_\infty) \tag{4}$$

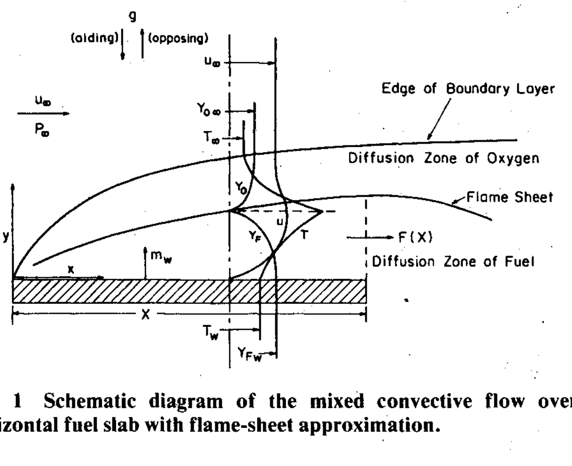

Fig. 1 Schematic diagram of the mixed convective flow over a horizontal fuel slab with flame-sheet approximation.

where

$$\Gamma_F=\frac{Y_F}{\nu_F W_F}-\frac{Y_O}{\nu_O W_O},\quad \Gamma_T=-\frac{c_p(T-T_\infty)}{Q}-\frac{Y_O}{\nu_O W_O}$$

are the Shvab-Zel'dovich coupling functions,[12] and P is the hydrostatic pressure given by

$$P=g\int_y^\infty(\rho-\rho_\infty)\mathrm{d}y' \tag{5}$$

The boundary conditions are

$$y=0:\qquad u=0,\quad T=T_w,\quad \lambda\frac{\partial T}{\partial y}=mL$$

$$Y_F=Y_{Fw},\quad mY_F-\rho D\frac{\partial Y_F}{\partial y}=m \tag{6a}$$

$$y\to\infty:\qquad u=u_\infty,\quad T=T_\infty,\quad Y_O=Y_{O\infty} \tag{6b}$$

$$y=y_f:\qquad Y_O=Y_F=0 \tag{6c}$$

where the subscript f designates the flame sheet.

To perform the boundary-layer transformation valid for the entire range of the mixed convective flow, we define a stream function

$$\psi(\xi,\eta)=(\sqrt{Re}/\xi^{1/10})f(\xi,\eta) \tag{7}$$

which satisfies the continuity equation through

$$\rho u=\mu_\infty\frac{\partial\psi}{\partial y}\quad\text{and}\quad \rho v=-\mu_\infty\frac{\partial\psi}{\partial x} \tag{8}$$

and the boundary-layer coordinates

$$\eta=\frac{\sqrt{Re}}{x\xi^{1/10}}\int_0^y\left(\frac{\rho}{\rho_\infty}\right)\mathrm{d}y,\quad \xi=(1+\zeta_h^2)^{-1} \tag{9}$$

where $\zeta_h=Gr/Re^{5/2}$ is the mixed convective ratio for the present horizontal case, while $Gr=g\beta(T_w-T_\infty)x^3/\nu_\infty^2$ and $Re=u_\infty x/\nu_\infty$ are the Grashoff and Reynolds numbers, respectively.

If we further normalize T and Γ_i by defining

$$\bar{T}(\xi,\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)},\quad \bar{\Gamma}(\xi,\eta)=\frac{(\Gamma_i-\Gamma_{i\infty})}{(\Gamma_{iw}-\Gamma_{i\infty})},\quad i=F,T \tag{10}$$

and also noting that

$$\eta=\frac{\sqrt{Re}}{x\xi^{1/10}}\int_0^y\left\{1-\frac{1}{\rho_\infty g}\frac{\partial P}{\partial y'}\right\}\mathrm{d}y'$$
$$=\frac{\sqrt{Re}}{x\xi^{1/10}}\left\{y-\frac{1}{\rho_\infty g}[P(x,y)-P(x,0)]\right\}$$

then Eqs. (2) and (3) can be transformed to

$$f''' + \frac{(6-\xi)}{10} ff'' + \frac{(\xi-1)}{5} (f')^2 \pm \sqrt{1-\xi}\frac{(4+\xi)}{10} \{ -\theta' [\eta+\delta\theta(\xi,0)] + \theta\} = \xi(1-\xi)\left\{f''\frac{\partial f}{\partial \xi} - f'\frac{\partial f'}{\partial \xi} \pm \sqrt{1-\xi}\left[\frac{\partial \theta}{\partial \xi} - \delta\theta'\frac{\partial \theta}{\partial \xi}(\xi,0)\right]\right\} \tag{11}$$

$$\bar{\Gamma}'' + Pr\frac{(6-\xi)}{10} f\bar{\Gamma}' = Pr\xi(1-\xi)\left\{\bar{\Gamma}'\frac{\partial f}{\partial \xi} - f'\frac{\partial \bar{\Gamma}}{\partial \xi}\right\} \tag{12}$$

where the (+) and (−) signs in Eq. (11) refer to aiding and opposing flows, respectively, and where we have used a prime (′) to designate $(\partial/\partial\eta)$. Furthermore, we also have $\delta = (T_w - T_\infty)/T_\infty$, $\theta = -\bar{P}(1+\zeta_h^2)^{1/10}/\zeta_h$, $\bar{P} = P/(\rho_\infty u_\infty^2)$ is the nondimensional pressure, and

$$\theta' = -\bar{T} \tag{13}$$

It may also be noted that unlike the case of mixed convective burning of a vertical surface, the use of the Howarth transformation does not reduce the system of equations to an incompressible form due to the presence of the normal pressure gradient here. The present equations do reduce to those obtained with the Boussinesq approximation by eliminating terms involving δ, which accounts for the density variations.

Because of the flame-sheet approximation, the temperature distribution is now given by

$$\bar{T}(\xi,\eta) = 1 + \ell(1-\bar{\Gamma}) \qquad \eta \le \eta_f$$
$$= [1+\ell(1-\bar{\Gamma}_f)]\bar{\Gamma}/\bar{\Gamma}_f \qquad \eta \ge \eta_f \tag{14}$$

where $\ell = (BL)/h_w$ and $B = [QY_{O\infty}/(\nu_O W_O) - h_w]/L$ is the transfer number. The flame location η_f is given by the requirement of $Y_O = Y_F = 0$ and the definition of $\bar{\Gamma}$. That is,

$$\bar{\Gamma}(\eta_f) = \sigma/(1+\sigma) \tag{15}$$

where $\sigma = (Y_{O\infty}\nu_F W_F)/(Y_{Fw}\nu_O W_O)$. Furthermore, since $Y_O \equiv 0$ when $\eta \le \eta_f$, it can be shown that the surface mass fraction Y_{Fw} is given by

$$Y_{Fw} = B/(1+\sigma+B) \tag{16}$$

It may also be noted that while $\bar{T}'$ and Y_i' are discontinuous at the flame, the quantities f, f', f'', θ, and θ' are continuous at η_f.

Thus the transformed boundary conditions for Eqs. (11) and (12) are,

$\eta = 0$: $\quad \bar{\Gamma} = 1, \quad f' = 0$

$$(6-\xi)f - 10\xi(1-\xi)\frac{\partial f}{\partial \xi} = \left(\frac{10B}{Pr}\right)\bar{\Gamma}' \tag{17a}$$

$$\eta \to \infty: \quad \bar{\Gamma} = 0, \quad f' = \xi^{1/5} \tag{17b}$$

The problem is thus completely defined: it is being governed by five parameters, namely ξ, B, σ, ℓ, and Pr. The parameter of special interest to the present study is the mixed convection parameter $\xi = (1 + Gr^2/Re^5)^{-1}$, which is bounded by 1 and 0 at the forced and free convective limits, respectively. Its physical significance can be interpreted in one of two ways. That is, if we first fix the streamwise location x, then increasing ξ implies either a decrease in the body force g or an increase in the freestream convective intensity u_∞. Alternatively, if we fix g and u_∞, then since $\xi \sim (1 + \text{const } x)^{-1}$, the boundary-layer structure is dominated by forced convection near the leading edge but will gradually change into one being dominated by natural convection as the flow moves downstream.

The practical parameters of interest to the present problem are the local mass burning rate

$$m_w = \rho_w v_w = -\left(\frac{B\lambda_\infty}{c_p}\right)\frac{\sqrt{Re}}{x\xi^{1/10}}\bar{\Gamma}'(\xi,0) \tag{18}$$

the fraction of the total pyrolyzate remaining unburned at ξ,

$$F(\xi) = \int_0^{y_f} \rho u Y_F \mathrm{d}y \Big/ \int_0^x m_w(x)\mathrm{d}x = Pr(1-\xi)^{1/4}\xi^{-7/20}\int_0^{\eta_f}[\Gamma(1+\sigma) - \sigma]f'\mathrm{d}\eta \div (1+\sigma+B)\int_1^\xi (1-\xi)^{-3/4}\xi^{-27/10}\mathrm{d}\xi \tag{19}$$

the wall shear stress

$$\tau_w = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0} = 2f''(\xi,0)(\rho u_\infty^2/2)/(\sqrt{Re}\xi^{3/10}) \tag{20}$$

and the flame standoff distance in the physical coordinate,

$$y_f = \frac{x\xi^{1/10}}{\sqrt{Re}}\int_0^{\eta_f}(1+\delta\bar{T})\mathrm{d}\eta \tag{21}$$

The final governing equations were solved using quasilinearization and iteration. The numerical method poses no particular difficulty in obtaining the solutions to the aiding flow. However, convergence problems have been encountered in obtaining a solution to the separating flows if the integration starts from the known solution of the pure forced convection limit. This difficulty has been resolved by replacing Eq. (11) by its differentiated form

$$f^{iv} + \frac{(6-\xi)}{10} ff''' + \frac{(3\xi+2)}{10} f'f'' \mp \sqrt{1-\xi}\frac{(4+\xi)}{10}\theta''[\eta+\delta\theta(\xi,0)] = \xi(1-\xi)\left\{f'''\frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial f''}{\partial \xi} \pm \sqrt{1-\xi}\left[\frac{\partial \theta'}{\partial \xi} - \delta\theta''\frac{\partial \theta}{\partial \xi}(\xi,0)\right]\right\} \tag{22}$$

with the additional boundary condition

$$f'''(\xi,0) = -\left(\frac{B}{Pr}\right)f''(\xi,0)\bar{\Gamma}'(\xi,0) \pm (1+\delta)\sqrt{1-\xi}\left\{\xi(1-\xi)\frac{\partial\theta(\xi,0)}{\partial \xi} - \frac{(4+\xi)}{10}\theta(\xi,0)\right\} \tag{23}$$

where the jump in θ'' can be calculated from Eq. (14) as

$$\frac{\theta''(\xi,\eta_f^-)}{\theta''(\xi,\eta_f^+)} = \frac{\ell\bar{\Gamma}_f}{(\ell\bar{\Gamma}_f-\ell-1)} \qquad (24)$$

Integration was carried out by first evaluating the solution to the governing equations with a given value of interest of $f''(\xi,0)$ at $\xi=1$ and then by marching in the negative direction of ξ until the boundary condition [Eq. (23)] was satisfied to an accuracy of 10^{-3}.

In addition to the solution of the horizontal case, we have also extended the previous calculations for the vertical case[10] to opposing flows up to the separation point. In this case the mixed convection parameter is $\xi=(1+\zeta_v^2)^{-1}$, where ζ_v is the corresponding mixed convection ratio Gr/Re^2.

Solutions for the rigorous formulation as well as those with the local similarity approximation were obtained; the latter case is defined by dropping all terms involving differentials with respect to ξ.

III. Results and Discussion

Since the system is characterized by five parameters, it is not feasible to conduct an extensive parametric study. Therefore, we shall study in detail the combustion response of a technologically important system, namely the mixed convective burning of polymethylmethacrylate (PMMA) in environments of different $Y_{O\infty}$. The associated physicochemical parameters are $Q=3244$ cal/g of oxidizer, $L=380$ cal/g of fuel, $T_w=660$ K, $W_F=100$, and $\nu_O/\nu_F=6$. Table 1 lists the values of the combustion parameters, $Y_{O\infty}$, B, and σ, used to generate the solution shown in Figs. 2-9. The value of the Prandtl number is taken as $Pr=0.7$, while the ambient temperature is fixed at $T_\infty=295$ K.

In Fig. 2 we present the results of velocity and temperature profiles as obtained from the rigorous numerical solution for different values of mixed convection ratio ζ_h, with $Y_{O\infty}=0.15$. Curve 2 corresponds to the forced convection limit whose velocity profile is that of the Blasius flow with blowing. For aiding mixed convective flows of curves 3-5, it is seen that the favorable pressure gradient induced by buoyancy accelerates the flow to approach the state corresponding to the free convection limit of curve 6. This flow acceleration steepens the velocity gradient and thereby increases the wall shear stress. Furthermore, the flame is moved closer to the wall and therefore the heat flux to the wall is also increased. Figure 2 also shows that because of the density reduction due to combustion, velocity overshoot develops with the increasing intensity of natural convection. Velocity overshoots in mixed convective flow have been both experimentally observed[14] and theoretically predicted[5] for small buoyancy-perturbed flows.

For opposing flows the buoyancy-induced pressure gradient retards the flow; curve 1 corresponds to the situation when the shear stress at the wall becomes zero and the flow separates. The flame is now farthest away from the wall, causing a reduction in the wall heat flux.

Table 1 Values of $Y_{O\infty}$, B, and σ for seven cases considered in this study (corresponding to PMMA burning in air[13])

$Y_{O\infty}$	B	σ
0.15	0.9905	0.1704
0.18	1.2428	0.1829
0.21	1.4951	0.1969
0.23	1.6632	0.2067
0.26	1.9155	0.2218
0.28	2.0837	0.2321
0.31	2.3360	0.2477

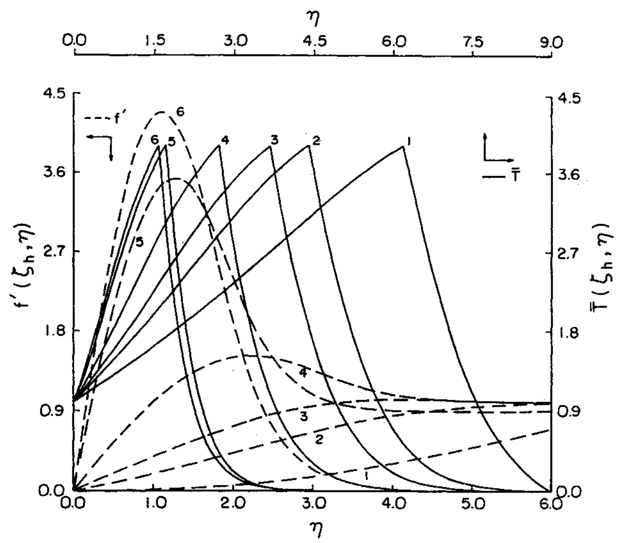

Fig. 2 Velocity and temperature profiles parameterized in ζ_h for horizontal PMMA slabs burning with $Y_{O\infty}=0.15$. Curves 1-6 designate $\zeta_h=-0.001$, 0.0, 0.007, 0.0707, 0.327, and ∞, respectively.

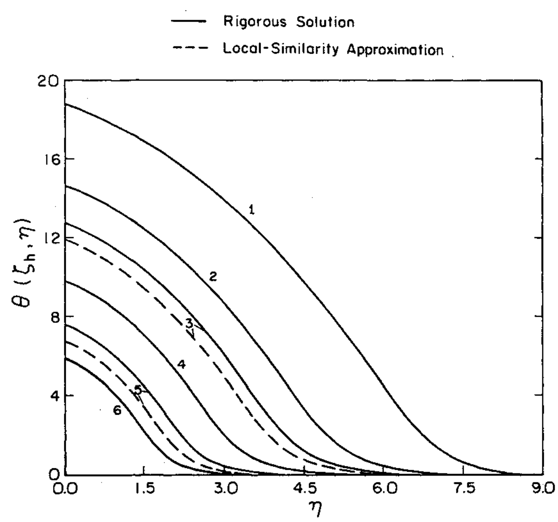

Fig. 3 Scaled pressure function $\theta(\zeta_h,\eta)$ for horizontal PMMA slabs burning with $Y_{O\infty}=0.15$. Curves 1-6 designate $\zeta_h=-0.001$, 0.0, 0.007, 0.0707, 0.326, and ∞, respectively.

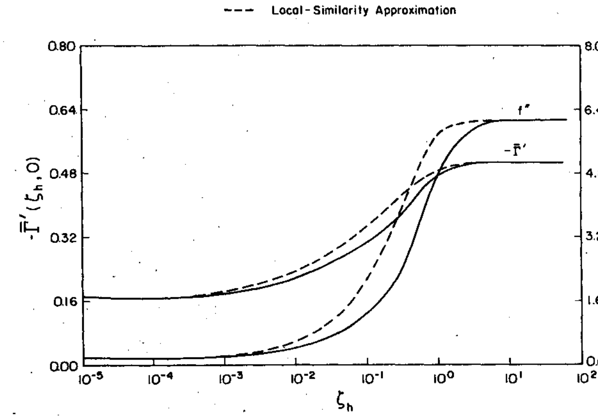

Fig. 4 Variations of surface shear coefficients $f''(\zeta_h,0)$ and scaled surface heat flux $\bar{\Gamma}'(\zeta_h,0)$ with ζ_h for horizontal PMMA slabs burning with $Y_{O\infty}=0.15$.

Figure 3 shows the scaled pressure function $\theta(\zeta_h,\eta)$. Since $\bar{P} \sim -\zeta_h\theta$, these results show that for aiding flows (curves 3-6) the buoyancy-induced pressure increases across the boundary layer from its initial value at the wall to the ambient pressure in the freestream. This induced pressure is the only force that counteracts the buoyancy force. For the purely forced convective flow (curve 2) $\bar{P}$ is of course zero, while for opposing flows (curve 1) it decreases from the wall value to the ambient condition in the freestream. Locally similar solutions are also presented for situations corresponding to curves 3 and 5 and show smaller values of θ.

In Fig. 4 we present the wall shear coefficient $f''(\zeta_h,0)$ and the scaled surface heat flux $\bar{\Gamma}'(\zeta_h,0)$ as functions of ζ_h. It is seen that both the shear stress and heat flux increase with an increase in the mixed convection ratio, with the intense mode of mixed convection occurring when $10^{-2} < \zeta_h < 10$. Results of the rigorous and local similarity solutions show maximum differences of 6% for $\bar{\Gamma}'(\zeta_h,0)$ and 34% for $f''(\zeta_h,0)$. Indeed, an important observation from the present study is that, while the local similarity solution approximates well the heat and mass transfer aspects of the flow, the differences are generally greater for the dynamical aspects of the problem.

Figure 5 shows the scaled flame-front standoff distance, $\bar{y}_f(\zeta_h) = \sqrt{Re}(1+\zeta_h^2)^{1/10}(y_f/x)$, and the excess pyrolyzate function $F(\zeta_h)$. The results show that with increasing buoyancy, the flame-front standoff distance decreases, while

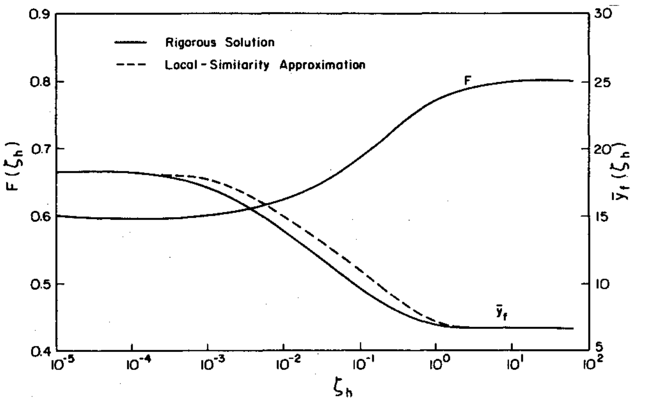

Fig. 5 Variation of scaled flame standoff distance $\bar{y}_f(\zeta_h)$ and the excess pyrolyzate function $F(\zeta_h)$ with ζ_h for horizontal PMMA slabs burning with $Y_{O\infty} = 0.15$.

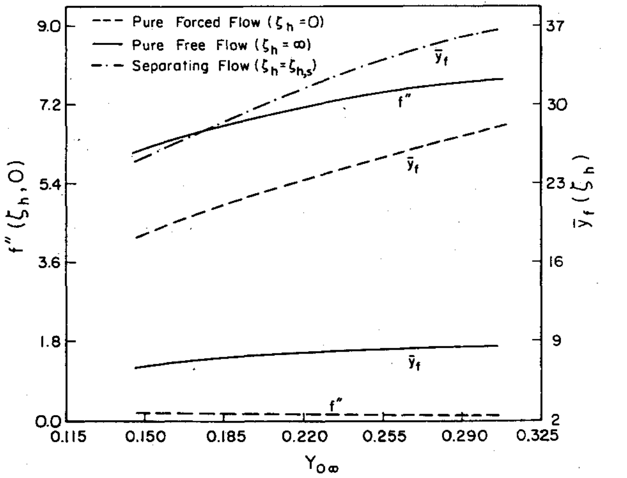

Fig. 6 Variations of scaled flame standoff distance $\bar{y}_f(\zeta_h)$ and surface heat flux $\bar{\Gamma}'(\zeta_h,0)$ with the oxidizer concentration for horizontal PMMA slabs.

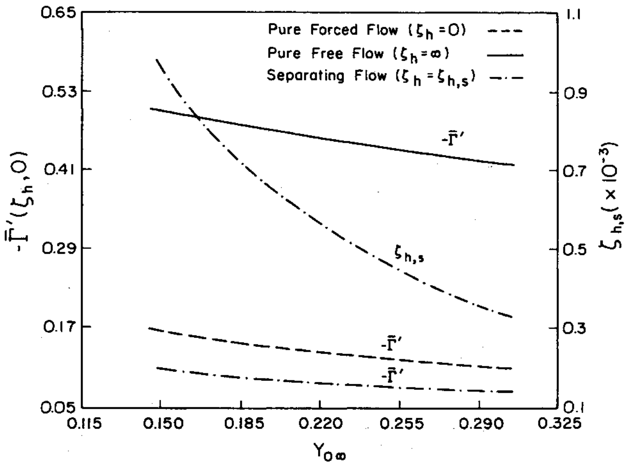

Fig. 7 Variations of the surface shear coefficient $f''(\zeta_h,0)$ and separation points $\zeta_{h,s}$ with oxidizer concentration for horizontal PMMA slabs.

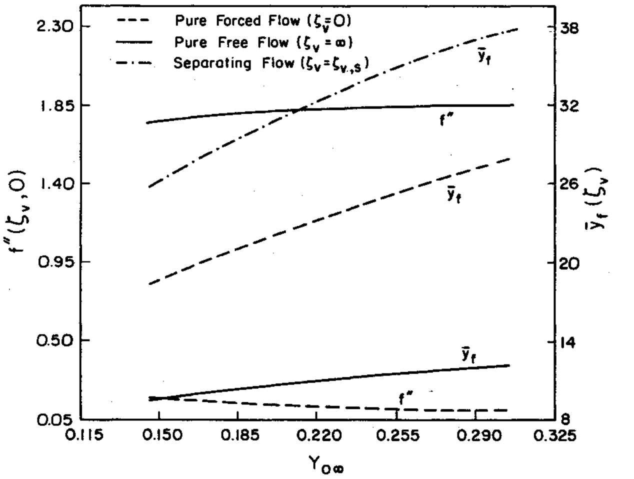

Fig. 8 Variations of scaled flame standoff distance $\bar{y}_f(\zeta_v)$ and surface heat flux $\bar{\Gamma}'(\zeta_v,0)$ with oxidizer concentration for vertical PMMA slabs.

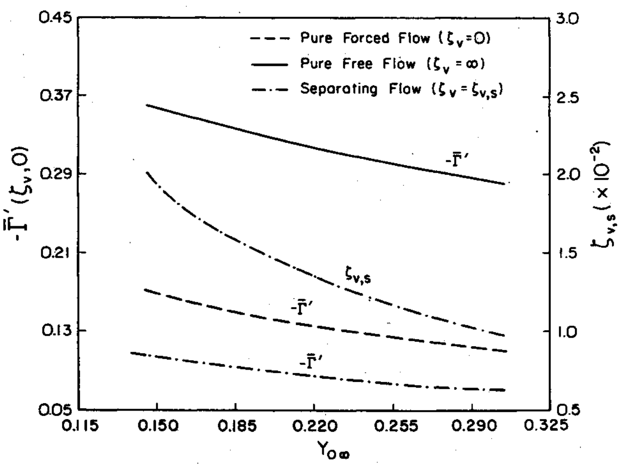

Fig. 9 Variations of the surface shear coefficient $f''(\zeta_v,0)$ and separation points $\zeta_{v,s}$ with oxidizer concentration for vertical PMMA slabs.

the excess pyrolyzate increases. The increase in pyrolyzate is quite interesting in that, even though y_f is reduced, the increase in the flow velocity has a stronger influence and thereby causes a net increase in the streamwise mass flow rate. This result has also been observed for mixed convective burning over a vertical surface.[6,10]

Figure 6 shows $\bar{y}_f$ and $f''(\zeta_h,0)$ for a range of the ambient oxidizer concentrations for the three distinctive cases of free, forced, and separating flows. It is seen that with increasing oxidizer concentration the flame moves away from the surface because of the increased surface blowing rate. For forced convective flow, the skin-friction coefficient decreases with increasing $Y_{O\infty}$ because of the increased blowing rate and thereby reduced surface shear. However, the tendency is quite the opposite for free convective flows. This is because the increased burning intensity accelerates the flow and therefore increases the surface shear.

Figure 7 shows the scaled heat flux $\bar{\Gamma}'(\zeta_h,0)$ and the separation location $\zeta_{h,s}$ as functions of $Y_{O\infty}$ for the cases of free, forced, and separating flows. Using the values of $-\bar{\Gamma}'(\zeta_h,0)$ here, it can be easily assessed that the mass burning rate, $m_w \sim -(B/c_p)\bar{\Gamma}'(\zeta_h,0)$, increases with $Y_{O\infty}$, as should be. The results also show that the separation points $\zeta_{h,s}$ lie within the range of 3×10^{-4}-10^{-3} for $0.15 \le Y_{O\infty} \le 0.31$.

Figures 8 and 9 correspond to Figs. 6 and 7 for the case of mixed convective burning over a vertical surface. The corresponding range of $\zeta_{v,s}$ is now between 10^{-2}-2×10^{-2}

IV. Conclusions

The introduction of the mixed convective parameter of the form $(Gr^2+Re^5)^{1/10}$ in the nondimensionalization of the governing equations, describing the steady, two-dimensional, chemically reacting laminar boundary-layer flows over a horizontal fuel slab, provides solutions that are uniformly valid over the entire range of mixed-flow intensities. The formulation of the governing equations is such that the pure forced and pure free convection are, respectively, recovered from the values of mixed convection number, $\xi = 1.0$ or 0.0. The governing equations become similar at both convective limits, as it should be. Comparison of the results from the local similarity method to those obtained from the rigorous solution show that the approximate method provides good agreement for the mass burning rate and physical flame standoff distances. With an increase in buoyancy, the wall shear stress and the excess pyrolyzate are found to increase, while the physical flame location from the wall decreases. The opposite effect is true for the opposing flows. For opposing flows, separation occurs when $2\times10^{-2} < \zeta_v < 10^{-2}$ and $3\times10^{-4} < \zeta_h < 10^{-3}$, respectively, for vertical and horizontal PMMA plates with $0.15 < Y_{O\infty} < 0.31$.

Acknowledgments

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